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THE BASAL KATABOLISM OF CATTLE AND OTHER SPECIES

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The basal katabolism of herbivora and especially of ruminants, unlike that of man or carnivora, cannot well be measured in the fasting state on account of the relatively large amount of feed always present in the alimentary canal of the former species. It may, however, be determined indirectly in the manner described by the authors[1,2,3] by measuring the total metabolism upon two different amounts of the same ration and from these data computing the level to which the metabolism would be reduced were all feed withdrawn. For example, a steer receiving two different amounts of the same mixed ration gave the following results:

	DRY MATTER EATEN DAILY	DAILY HEAT PRODUCTION
	kgms.	*calories*
Period 2	9.146	16,511
Period 1	4.463	10,905
Difference	4.683	5,606
Heat increment per kilogram of dry matter		1,197

Evidently, out of the total metabolism of 10905 Calories in Period 1, 1197 $\times$ 4.463 = 5342 Calories may be regarded as the heat production caused by the 4.463 kgm. of dry matter eaten while the remainder, 5563 Calories is the basal katabolism.

Our investigations upon the metabolism of cattle, which have been published elsewhere[3] afford data for computing in the manner just illustrated the basal katabolism of ten unfattened steers in twenty-seven experiments. In view of the very striking effect of standing in increasing the metabolism of cattle the

basal katabolism per 24 hours has been computed separately from the observed rate of heat production during the intervals of lying and standing, respectively, and also for 12 hours standing and 12 hours lying per day, assumed as representing average conditions.

As was to be expected, the basal katabolism increased with the size of the animal but with very considerable fluctuations. The graphs of the results indicate an equally close relation of the basal katabolism with the weight and with the two-thirds power of the weight (computed body surface) and this conclusion is confirmed by a comparison of the coefficients of correlation as follows:

Coefficients of correlation

	WITH LIVE WEIGHT	WITH 2/3 POWER OF LIVE WEIGHT
Basal katabolism, lying 24 hours	0.8655 ± 0.0326	0.9032 ± 0.0239
Basal katabolism, standing 12 hours	0.8733 ± 0.0308	0.8710 ± 0.0313
Basal katabolism, standing 24 hours	0.8548 ± 0.0350	0.8250 ± 0.0415

Computing the basal katabolism per square meter of body surface as estimated by Moulton's formulae[4] viz.,

For unfattened animals $S = 0.1186\ W^{5/8}$

For fattened animals $S = 0.158\ W^{5/9}$

the following results were obtained.

Basal katabolism of cattle per square meter of body surface

	LYING 24 HOURS	STANDING 12 HOURS	STANDING 24 HOURS
Mean, Calories	964.0	1173.0	1365.0
Probable error of mean, Calories	± 24.0	± 21.4	± 25.7
Probable error of single result, Calories	±124.8	±110.9	±133.6
Standard deviation, Calories	185.1 ± 17.0	164.5 ± 15.1	198.0 ± 18.2
Coefficient of variability	0.1920	0.1462	0.1451

A positive correlation of the basal katabolism per square meter body surface with the live weight was also found as follows:

Coefficients of correlation with live weight

Basal katabolism per square meter

Lying 24 hours	0.5375 ± 0.0923
Standing 12 hours	0.3666 ± 0.1124
Standing 24 hours	0.2405 ± 0.1223

The results show the marked influence of standing upon the metabolism of cattle, the mean 24 hour basal katabolism lying, standing 12 hours and standing 24 hours being in the proportion of 100 : 121 : 141, the differences largely exceeding the probable errors. Computing, from the results per square

meter of surface, the basal katabolism for 12 hours standing and 12 hours lying gives as the maintenance requirement for a 1000-pound steer 5918 ± 560 Calories.

The results for the basal katabolism of man reported by Benedict, Emmes, Roth and Smith,[5] and by Means[6] present much the same picture as ours upon cattle with the exception of a much lower variability.

Coefficients of correlation

	WITH BODY WEIGHT	WITH BODY SURFACE
Total basal katabolism		
98 men	0.7263 ± 0.0320	0.7747 ± 0.0272
75 women	0.7759 ± 0.0310	0.7447 ± 0.0347

Daily basal katabolism of men and women per square meter of surface

	MEN	WOMEN
Mean Calories	830.0	768.0
Probable error of mean, Calories	± 4.3	± 4.9
Probable error of single result, Calories	±42.3	±42.8
Standard deviation, Calories	62.7 ±3.0	63.5 ±3.1
Coefficient of variability	0.0755	0.0827

Correcting for the error shown by D. and E. F. DuBois[7] to be incident to the use of the Meeh formula, the means for men and women are as follows:

Corrected daily basal katabolism of men and women per square meter of body surface

	MEN	WOMEN
Means, Calories	935.0	886.0
Probable error of mean	± 4.8	± 5.8
Probable error of single result	±47.5	±49.4

Including the data obtained by Meissl, Strohmer, and Lorenz[8] by Tangl[9] and by Fingerling, Köhler and Reinhardt[10] for swine and by Zuntz and Hagermann[11] for the horse, the following comparison of species may be made.

Mean daily basal katabolism per square meter of body surface

Men (complete muscular rest)	935 ± 5
Women (complete muscular rest)	886 ± 6
Cattle (lying)	964 ± 24
Hogs (lying)	1078 ± ?
Horse (standing quietly)	948 ± ?

Considering the nature of the results they show a rather striking degree of uniformity and tend to confirm the conclusions of E. Voit[12] that the basal

katabolism of different species of animals is substantially proportional to their body surface. It may be surmised that the exceptional result with the hog is due to the imperfect data available for computing the body surface of this species.

[1] Armsby, *Washington, D. C., U. S. Dept. Agric., Bur. Anim. Indust., Bul.* **142,** 1912.
[2] Armsby and Fries, *Ibid., Bul.* **128,** 1911.
[3] Armsby and Fries, *J. Agric. Res., Washington,* 3, 1915, (435); **10,** 1917, (599); **11,** 1917.
[4] Moulton, *J. Biol. Chem., New York,* **24,** 1916, (299).
[5] Benedict, Emmes, Roth and Smith, *Ibid.,* **18,** 1914, (139).
[6] Means, *Ibid.,* **21,** 1915, (263).
[7] DuBois and DuBois, *Arch. Inter. Med.,* **15,** 1915, (868).
[8] Meissl, Strohmer and Lorenz, *Zs. Biol., München* **22,** 1886, (63).
[9] Tangl, *Biochem. Zs.,* **44,** 1912, (252).
[10] Fingerling, Köhler and Reinhardt, *Landw. Versuchstat, Berlin,* **84,** 1914, (149).
[11] Zuntz and Hageman, *Landw. Jahrb., Berlin,* **27,** 1898, Ergzbd. III, (284).
[12] Voit, E., *Zs. Biol., München,* **41,** 1901, (113).

THE LOCATION OF THE SUN'S MAGNETIC AXIS

By F. H. Seares, A. van Maanen, and F. Ellerman

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Communicated by G. E. Hale, November 26, 1917

The discovery by Mr. Hale in 1913 of a general magnetic field[1] surrounding the sun raised at once questions of great interest. First among these was the character of the field and the variation of its intensity over the solar surface. A preliminary investigation showed that approximately the sun may be regarded as a uniformly magnetized sphere, its axis coinciding with the axis of rotation. The minuteness of the observed quantities and the difficulties experienced in their measurement necessitated the provisional acceptance of this simple hypothesis; nevertheless, well-known peculiarities of the earth's magnetic field suggested that the solar field might deviate from that of a spherical magnet, and that its axis might be inclined to the rotation axis by an amount susceptible of measurement by special series of observations. This communication is concerned with the latter of these questions, namely, the position of the magnetic axis.

Observations of the sun's field are made by placing the slit of the spectrograph in coincidence with the central solar meridian. A compound quarter-wave plate and a Nicol prism just outside the slit serve as an analyzer, the observed effect being a minute displacement of an appropriately chosen spectral line. The amount of the displacement varies with the inclination of the lines of force to the line of sight, in other words, with the position of the sun's magnetic axis, the heliographic latitude of the point observed, and the dis-

tance of the observer from the plane of the sun's equator. If the field be that of a uniformly magnetized sphere,[2]

$$k\Delta = \{ 3 \sin (2\phi - D) + \sin D \} \cos i + \{ 3 \cos (2\phi - D) + \cos D \} \sin i \cos \lambda \quad (1)$$

in which

Δ = displacement of spectral line;
ϕ = heliographic latitude of point observed;
D = angular deviation of observer from plane of sun's equator;
i = inclination of sun's magnetic axis to axis of rotation;
λ = heliographic longitude of north magnetic pole referred to central meridian;
k = constant depending on the units and the behavior of the line in a field of known intensity.

For $i = 0$, equation (1) reduces to

$$k\Delta = 3 \sin (2\phi - D) + \sin D, \quad (2)$$

and, if D also is zero, to

$$k\Delta = 3 \sin 2\phi. \quad (3)$$

Since the maximum value of D is about 7°, equation (2) differs but little from (3). The displacement curves derived by Mr. Hale from preliminary observations agreed substantially with these equations, whence it follows that i must also be small and that the difference between equations (1) and (2), which represents the influence of i, is a quantity of the second order.

When Δ is expressed in thousandths of a millimeter, k, for the lines observed, is of the order of unity. The maximum displacement, by equation (3), is therefore 3 or 4 μ (about 0.001 A). To determine the position of the magnetic axis, quantities of the order of 0.5 μ must accordingly be evaluated. This indicates sufficiently the nature of the problem and the degree of precision that had to be attained. It was evident from the beginning that a long and carefully executed series of observations would be required for a successful attack on the problem.

The original investigation by Mr. Hale was based on only four lines. Later observations have increased the number known to be affected by the sun's field to 30, for 18 of which results were communicated at the Atlanta meeting of the American Astronomical Society in December, 1913. For the investigation here described three chromium lines, $\lambda\lambda$ 5247, 5300, 5329 were selected, which are of special suitability for measurement because of intensity (2 and 3), location in the spectrum, and magnitude of displacement.

From June 8 to September 23, 1914, these lines were photographed daily under the direction of Ellerman, with almost no break in the series. The circumstances were most favorable owing to the small number of sun-spots, whose magnetic fields, many times the intensity of the underlying field of the sun, seriously complicate the investigation. Because of advantages connected

with the numerical solution and the necessity of limiting what at best could be only a very laborious undertaking, the observations were confined to the zone 45°N–45°S. Twelve spectrograms with exposures of 10 to 30 minutes constituted the normal observing program for each day. For 63 of the days the photographs have been completely measured by van Maanen, who has assumed the responsibility for this part of the undertaking. More than 2000 sets of measures were required, each involving about a hundred settings of the micrometer.

In measures of minute displacements of spectral lines, systematic errors are always to be suspected, as well as the influence of prejudice arising from a knowledge of the results that will satisfy a given hypothesis. Such systematic errors as may have entered in the present case probably affect only the constant k, which varies from line to line but does not enter into the determination of the position of the magnetic axis.

To exclude the influence of prejudice, the procedure devised by Mr. Hale has been followed here. The limited zone of heliographic latitude covered by a single spectrogram may lie in the northern hemisphere, where the displacements are positive, or in the southern hemisphere, where they are negative; or it may extend over the equator and thus show only very small displacements, some negative and some positive. The measurer has rarely known in advance the latitudes covered by any spectrogram. Further, the above distribution of algebraic signs presupposes that the photograph has been made with the compound quarter-wave plate in its normal position. Since the inversion of the plate reverses the signs of the displacements, its position, as a final precaution, has been varied at random by the observer, and the measurer has not known the position used for a given photograph until after his settings were finished.

The data have been treated as follows: Each displacement affords an equation of condition of the form (1) for the determination of the unknowns k, i, and λ. The longitude of the magnetic pole, λ, involves an epoch, t_0, when the pole was on the central meridian, and the period, P, in which the magnetic axis revolves around the axis of rotation. For a single day we may assume λ to be constant, which leads us to discuss separately the observations for each day and for each line, thus deriving values for two new unknowns, x and y, which are functions of k, i, and λ. The analysis of x and y for the whole series of days then determines k, i, P, and t_0.

Equation (1) may be written

$$Ax + By = \Delta. \tag{4}$$

A and B are the bracketed expressions of (1), including only known quantities, and

$$x = k^{-1} \cos i, \qquad y = k^{-1} \sin i \cos \lambda \tag{5}$$

whence

$$Y = y/x = \tan i \cos \lambda. \tag{6}$$

About 50 values of Δ were available for each line on each day. Means were found for groups of 5 or 6 adjoining displacements, thus giving 8 or 10 observation equations of the form (4) for a least-squares determination of x and y.

The individual values of Δ for September 2, 1914, a series of average weight, are plotted in Fig. 1 against the latitudes as abscissae. The close agreement with a sine curve of the type of equation (3) appears at a glance. The calculated displacement-curves corresponding to the values of x and y derived from these data, are also shown in the figure. Their ordinates for $\phi = 0$, namely, $+0.8$, $+1.0$, and $+0.5$ μ, respectively, are of the order of the small quantities

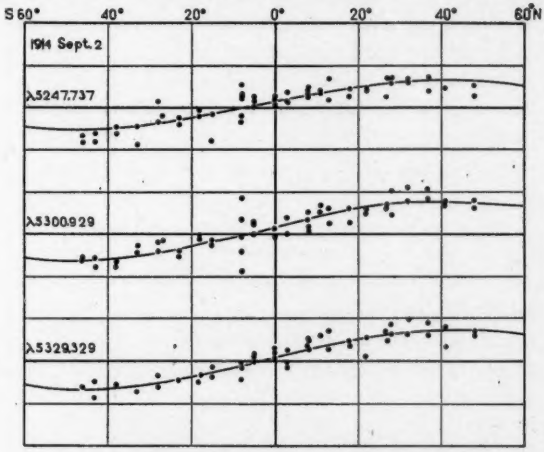

FIG. 1. DISPLACEMENT-CURVES FOR 1914, SEPTEMBER 2

Abscissae are heliographic latitudes. Ordinates are displacements, the scale being 1 division of diagram = 0.005 mm. The curves, which correspond to equation (1), have been derived from the observed values of Δ. Their ordinates for $\varphi = 0$ represent the combined influence of k, D, i, and λ. These data for the three lines give $Y = \tan i \cos \lambda = +0.213$ which is plotted as a single point in Fig. 2, together with similar values of Y for each of the other dates.

which differentiate the displacement-curve (1) from the curve (2) and indicate the precision with which the curves must be located in order to determine the value of i.

Having found x and y from each line for each day, the results were combined by (6) to form weighted mean values of Y, which were then plotted with the times as abscissae. These should define a sine curve whose amplitude and period are, respectively, tan i and P. The individual points are reproduced in Fig. 2, from which approximations for i, P and t_0 were easily derived. The final values and their probable errors

$$i = 6^\circ.2 \pm 0^\circ.4, \qquad P = 31.79 \pm 0.31 \text{ days}$$
$$t_0 = 1914, \text{ June } 25.31 \pm 0.42 \text{ days, G. M. T.}$$

were calculated by a least-squares solution giving differential corrections to the approximate values read from the curve.

The theoretical curve for $\tan i \cos \lambda$ corresponding to these elements is also shown in Fig. 2. So far as accidental errors are concerned, the representation is very satisfactory, for the amplitude of the curve corresponds to an extreme difference of only $1\,\mu$ in the position of the individual displacement curves. There is some evidence of a gradual increase in $\tan i$; but no emphasis is placed on the phenomenon. Whether it is real or whether it is due to errors of measurement and the influence of spots and the 'dark markings' of the Greenwich observers cannot now be determined. Days on which spots were near the central meridian were generally avoided. The magnetic fields surrounding some of the 'dark markings' noted at a later time (our attention was first directed to them in 1915) were measured and found to be two or three times as intense as the sun's general field; but their existence and influence upon the measures here discussed cannot now be traced.

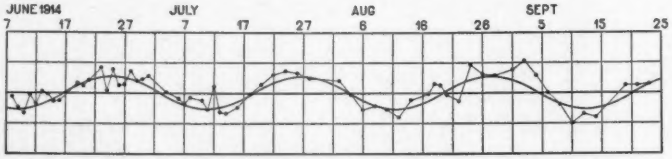

FIG. 2. THE CURVE $Y = \tan i \cos \lambda$

Each plotted point is derived from data for a single day similar to that illustrated in Fig. 1. Approximate values of i, P, and t_0 were read from a provisional curve. Differential corrections derived by a least-squares solution gave the final values, which correspond to the curve shown in the figure.

The result for P is of much interest in relation to the rotation period of the sun. For the reversing layer this ranges from 26.4 days (synodic period) at the equator to about 30.5 days at 45°, the value for P not being equalled until approximately 55° is reached. Hence for latitudes lower than this limit, the magnetic axis in its motion about the axis of rotation appears to lag behind the reversing layer. The bearing of this circumstance upon the physical constitution of the sun must await further elucidation. In the meantime, however, it should be noted that P may not be constant. The present series of measures, at any rate, is insufficient to establish its freedom from fluctuations; and none of the earlier observations, which were made with other purposes in view, are adapted to the investigation.

The fact that the observed values of Y agree satisfactorily with the periodic function $\tan i \cos \lambda$ does more than show that i is not zero; it affords a most convincing demonstration of the existence of the magnetic field itself. The measures of each of the three lines, on each of the sixty odd days, define a displacement-curve which is an independent confirmation of the existence and general character of the field; but the fact that the curves for the separate days are

so related to each other as to satisfy equation (6) increases enormously the weight of the conclusion and seems to exclude the possibility that it should be the consequence of an obscure and unsuspected systematic error.

We desire to express our great obligation to Miss Wolfe of the Computing Division who has executed most efficiently the numerous least-squares solutions required for the discussion of the data.

[1] Hale, G. E., *Terr. Mag., Baltimore*, **17**, 1912 (173–178); *Mt. Wilson Contr.* No. 71, *Astroph. J., Chicago*, **38**, 1913, (27–98).

[2] Seares, F. H., *Mt. Wilson Contr.* No. 72, *Astroph. J., Chicago*, **38**, 1913, (99–125).

RESONANCE AND IONIZATION POTENTIALS FOR ELECTRONS IN CADMIUM, ZINC, AND POTASSIUM VAPORS[1]

By John T. Tate and Paul D. Foote

University of Minnesota and Bureau of Standards

Communicated by R. A. Millikan, December 19, 1917

It has been shown by Franck and Hertz and others that there are, for electrons accelerated through gases or vapors, certain definite potentials at which there is a large transfer of energy from the electron to the atom, as evidenced by the emission at these potentials of radiations characteristic of the gas atom. This is to be expected on purely mechanical grounds since no considerable transfer of energy from the light electron to the relatively heavy gas atom can take place except when the time of encounter between electron and atom bears some simple relation to the characteristic period of one of the vibrational degrees of freedom in the atom. It is therefore to be expected that there will be a critical potential corresponding to each absorption line of the gas and that at this potential the electrons will give up their energy to the gas and cause the emission of a radiation of the frequency of the corresponding absorption line.

Two types of inelastic encounter between electrons and gas atoms have been observed. One of these results in the emission of a radiation of a single frequency, without ionization of the gas, while the other ionizes the gas and causes it to emit a composite spectrum of radiations. The potential giving the first type of encounter may be termed a resonance potential, that giving the second type an ionization potential.

The present paper is an account of an experimental determination of the resonance and ionization potentials in cadmium zinc, and potassium vapors.

The method employed was that described by Tate[2] for the determination of critical potentials in mercury vapor and the apparatus was similar to that used by us[3] in our work on sodium vapor.

The results obtained are shown in the accompanying table which also contains for the sake of completeness the results, already published, for sodium vapor.

METAL	RESONANCE POTENTIAL (VOLTS)		WAVE LENGTH OF RADIATION	IONIZATION POTENTIAL (VOLTS)		LIMITING WAVE LENGTH OF SERIES
	Observed	Calculated	Å	Observed	Calculated	Å
Cadmium	3.88	3.79	3260.17	8.92	8.97	1378.69
Zinc	4.10	4.02	3075.99	9.50	9.37	1319.95
Potassium	1.55	1.65	7685.0	4.10	4.33	2856.0
Sodium	2.12	2.10	5893.0	5.13	5.13	2413.0

The curves for zinc vapor indicate the possible presence of a secondary inelastic encounter, apparently without ionization, at 2.3 volts; but further work is in progress to make sure of this point.

It will be seen that in all cases the results agree within the limits of experimental error with the values as calculated from the quantum relation $h\nu = eV$, where ν is the frequency of the single radiation in the case of resonance potentials or the limiting frequency of the series of radiations in the case of ionization potentials.

It is to be expected that the frequencies corresponding to the ionization potentials should be the long wave-length limit of the photo-sensibility of the vapor in question. Experiments are now in progress to determine whether or not this is so.

Concerning the question of the cause for the appearance of ionization in metallic vapors at applied potentials much lower than the ionization potential it should be remarked that in no case was there an observable decrease in the value of the resonance potential even under conditions which permit of the appearance of the complete spectrum at low potentials. It is therefore thought that this setting in of ionization at applied potentials much lower than the ionization potential is due rather to the presence of electrons having high initial velocities than to a decrease in the value of the critical potentials. The curves obtained with potassium vapor clearly show the presence of a large number of electrons having initial velocities corresponding to 1.6 volts.

[1] Abstract of a paper presented before the American Physical Society, December 1, 1917.

[2] Tate, *Physic. Rev. Ithaca, N. Y.*, **10**, 1917, (81).

[3] Tate and Foote, *Washington, J. Acad. Sci.*, **7**, 1917, (517).

THE VALIDITY OF THE EQUATION $P = T\ dv/dT$ IN THERMO-ELECTRICITY

By Edwin H. Hall

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Read before the Academy, November 21, 1917

The equation $P = T\ dv/dT$, in which P stands for the Peltier effect and v for the Volta effect between any two metals, was first obtained, I believe, by Lord Kelvin, comparatively late in his life. He derived it from a course of argument involving the imagined performance of a certain isothermal cycle. It was derived later by O. W. Richardson through a series of steps including his theory of thermionic emission and the imagined performance of a reversible cycle taking electrons from a metal at one temperature and putting them back into the same metal at a different temperature.

When Kelvin came to test his formula by the experimental data available to him he found no support for it. In fact, it is said that "the experimental verification failed by a thousandfold."

During the last few years experiments made by various skillful investigators measuring dv/dT in high vacua have perhaps reduced the ratio of the two sides of the professed equation to fifty; but it still remains so large that it would be considered a complete refutation of the equation's pretensions, if it were not possible to find an escape from this conclusion by dwelling upon the great difficulty of the experimental investigation.

I have not, until recently, given much attention to this equation; but two or three months ago my colleague, Professor Bridgman, who has lately been studying thermo-electricity intensively, put into my hands for criticism a manuscript in which, going over with modifications of his own the Kelvin method and the Richardson method, he had in both cases reached their conclusion. Studying Bridgman's manuscript I am, on the contrary, strongly inclined to the opinion that in each method the proof fails. As Professor Bridgman is now too much occupied with government work to give adequate attention to the question at issue, he has given me permission to present this question here.

I cannot reproduce at length either of the arguments. I can only point out what I believe to be the weak link in each. In the first, which was suggested by that of Kelvin, Bridgman assumes that when, by an applied e.m.f., electricity is carried along a metallic path from metal A, constituting one plate of a condenser, to metal B, constituting the other plate, the only heat phenomenon, except the negligible resistance effect, is the production or absorption of heat at the junction of the two metals, the ordinary Peltier effect. This assumption, which is vital to the argument, I question.

Let n_e be the number of free electrons per unit volume of a metal and n_i

the number of positive ions, atoms lacking each an electron. Within the interior of a metal in electrical equilibrium we must have $n_e = n_i$. At the surface, if the metal has a static charge, we do not have such equality. Now, according to my view, the free electrons and the positive ions, which are the products of the ionization of the atoms, obey the mass-law, so that everywhere in a metal we have

$$n_e \times n_i = a, \textit{ constant.}$$

If this view is sound, when we take electrons from one plate of a condenser and convey them to the other plate, we thereby disturb the electrical equilibrium in each metal. If one metal loses, for example, q electrons, we cannot have mass-law equilibrium in this metal until sufficient new ionization occurs therein to make the total number of free electrons if this metal only $\frac{1}{2}\ q$ less, and the total number of positive ions $\frac{1}{2}\ q$ more, than at first. In the other metal the converse operation must take place, re-association occurring there until the number of free electrons is $\frac{1}{2}\ q$ more and the number of positive ions $\frac{1}{2}\ q$ less than before. These processes of ionization and of re-association would balance each other in heat production if the two plates of the condenser were of the same metal, but otherwise they do not. Of course, wherever in the metals the ionization and the re-association primarily occur the result will presently appear at the surface, since any excess or deficiency of electrons in the interior of a metal must correct itself at the expense of the surface. Practically, then, we may regard the ionization and the re-association as occurring at the surface only.

Bridgman says that the failure of the equation $P = T\ dv/dT$ to bear the experimental test led Kelvin "to infer the existence of reversible heating effects at the surface of a metal when a charge is added to or subtracted from the surface, just as he had previously inferred the existence of the Thomson heat." Bridgman has thought that this inference was not justified and that sufficiently careful experiments would verify the equation. I think, on the contrary, that Kelvin's inference was sound, that the ionization and re-association phenomena which I have indicated are precisely the reversible heating effects at the surface which he infers, but does not visualize, and that the equation in question will never be justified by experiment.

The other line of argument begins with a metal in equilibrium with a surrounding electron atmosphere contained within an enclosure otherwise vacuous. The following quotations are from the second chapter of Richardson's *Emission of Electricity From Hot Bodies:*

"In this steady state there will be a definite number n [of electrons] per unit volume, on the average, in the vacuous enclosure, and they will exert a definite pressure p. If the enclosure is provided with a cylindrical extension in which an insulating piston can move backwards and forwards, this pressure p can be made do work against an external force." "The relation between the pressure of these electrons and the temperature of the enclosure can be found

by an application of the second law of thermodynamics." "All we need is an expression for dS, the increment in the entropy caused by motion of the piston. If φ is the change in the energy of the system which accompanies the transference of each electron from the hot body to the surrounding enclosure, then

$$dS = \frac{1}{T}[d(n\ v\ \varphi) + pdv],\text{" etc.}$$

In writing this equation, which is fundamental to his argument, Richardson treats the case of a metal emitting electrons precisely as one treats the case of a body of water giving off steam to push against a piston. That is, he treats the emission of electrons as a process strictly comparable with evaporation. But there is an important difference between the two processes. In evaporation the thing given off is of the same substance as that left behind. In the emission of electrons this is not true. Evaporation leaves the constitution of the remaining liquid unchanged. Emission of electrons continually changes the constitution of the emitting metal, unless other electrons are put into the body to make good the loss. When a body emits a certain mass m of electrons under the conditions described by Richardson, the system under discussion takes in something more than heat energy; it takes in *substance*, the mass m of electrons. There is no analogue to this in the process of evaporation, and it remains to be shown that the equation which I have quoted from Richardson, an equation that holds beyond question for the case of evaporation, holds also for the case of emission of electrons.

One cannot, according to my view, meet this difficulty by supposing the body of metal made very large, so large that the static charge produced on it by the emission of a mass m of electrons without compensation would be negligibly small. For, if the loss of the electrons is not made good, the mass-law, requiring that $n_e \times n_i$ shall remain constant within the metal, will cause ionization there proportional to m, without regard to the amount of the metal; and this ionization will introduce a consumption of heat for which Richardson has made, I think, no provision.

ON THE EQUATIONS OF THE RECTANGULAR INTERFEROMETER

By Carl Barus

Department of Physics, Brown University

Communicated December 8, 1917

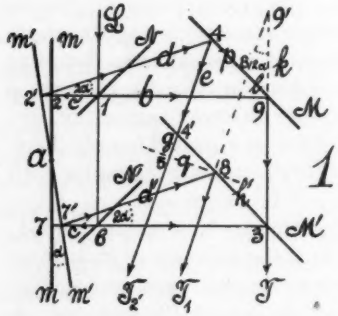
1. Auxiliary Mirror.—It is desirable to deduce the fundamental equations more rigorously than has heretofore[1] seemed necessary. Figure 1 is supplied for this purpose, and represents the more sensitive case, where in addition to the mirrors M,M', N,N' (all but M being necessarily half silvers), there is an

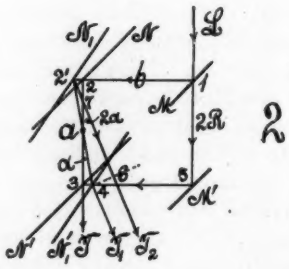
auxiliary mirror mm, capable of rotation (angle α) about a vertical axis A. The mirrors M–N' in their original position are conveniently at 45° to the rays of light,, while mm is normal to them. Light arriving at L is thus separated by the half silver N at 1, into the two components 1,2,1,9,3,T and 1,6,7,6,3,T, interfering in the telescope at T.

When mm is rotated over a small angle α, these paths are modified to 1,2,2′,4,4′,5,T_2 and 1,6,7′,8,T_1. T_1 and T_2 enter the telescope in parallel and produce interferences visible in the principal focal plane, provided the rays T_1 and T_2 are not too far apart, in practice not more than 1 or 2 mm. Interference fringes therefore will always disappear if the angle α is excessive, but the limits are adequately wide for all purposes. The essential constants of the apparatus are to be

$$(9,1) = (6,3) = b; \quad (1,2) = (6,7) = c; \quad (9,3) = (1,6) = 2R,$$

R being the radius of rotation.





When the mirror mm is rotated to $m'm'$ over the angle α, the new upper path will be

$$c + R \tan \alpha + d + e + g,$$

where

$$(2',4) = d, \ (4,4') = e, \ (4',5) = g,$$

the plane $(8,5) = q$ normal to T_1 and T_2 being the final wave front. The lower path is similarly $2R + (c - R \tan \alpha) + d'$ to the same wave front (8,5) where $(7',8) = d'$. Hence (apart from glass paths which have been treated, the path difference $n\lambda$ (n being the order of interference) should be

$$n\lambda = 2R (\tan \alpha - 1) + d - d' + e + g.$$

The figure in view of the laws of reflection then gives us in succession

$$d = (b + c + R \tan \alpha)/(\cos 2\alpha + \sin 2\alpha),$$
$$d' = (b + c - R \tan \alpha)/(\cos 2\alpha + \sin 2\alpha),$$
$$e = 2R/(\cos 2\alpha + \sin 2\alpha),$$
$$g = 2R \sin 2\alpha (1 + \tan \alpha) (\cos 2\alpha - \sin 2\alpha)/(\cos 2\alpha + \sin 2\alpha),$$
$$q = 2R \sin 2\alpha (1 + \tan \alpha).$$

To obtain g it is sufficient to treat the similar triangles (3,8,9′) and (9,8′,9′) where $h = (9,4)$, $h' = (3,8)$, $k = (9,9')$, $l = (9,8')$ may be found in succession as the normal distance between the mirrors M and M' is $R\sqrt{2}$, so that finally

$$g = (h - l) \sin (45° - 2\alpha), \quad q = (h - l) \cos (45° - 2\alpha).$$

If these quantities are introduced into the above equation for $n\lambda$ we may obtain after some reduction

$$n\lambda = 4 R \sin \alpha (\cos \alpha - \sin \alpha).$$

Since $n\lambda = 2\Delta N \cos i$, ΔN being the normal displacement of the mirror M', and $i = 45°$, the corresponding equation to the second order of small quantities α is

$$\Delta N/\Delta\alpha = 2 R (\cos \alpha - \sin \alpha)/\cos i = 2\sqrt{2} R (1 - \alpha - \alpha^2/2).$$

If α is sufficiently small, the coefficient is simply $2 R/\cos i$ as used heretofore.

There remain the glass paths which for the rays d and d' are compensated. Additionally the upper ray has a glass path 3 displaced to 4′. The lower ray has the fixed path at 1, and this is equal to the other at 1, since the angles are 45°. Thus the variable part of the glass paths at 3 to 4′ is uncompensated and the angle of incidence changes from 45° to 45° − 2α. The reflecting sides of the plates are silvered. Hence $e (\sin i - \cos i \tan r) 2\Delta\alpha$ must be added to the equation.

2. Rotating Doublet.—The second case, figure 2, in which the auxiliary mirror of the preceding apparatus is omitted is, curiously enough, inherently simpler. M,M', N,N', are mirrors, half silvered at (1) and (3) and the two latter on a vertical axis A and rigidly joined by the rail (2,3). The mirrors being preferably at 45°, the component rays are 1,2,3,T and 1,5,3,T, the mirror M' being on a micrometer with the screw normal to the face. The ray parallelogram is made up as before of $(1,2) = b = (3,5)$ and $(1,5) = 2 R = (2,3)$. When the rail (2,3) is rotated over an angle α, the mirrors take the position N_1 and N_1' at an angle α to their prior position and the angle of incidence is now 45° − α. The new paths, if (4,6) is the final wave front, are thus (1,2,2′, 6,T_2) and (1,5,4,T_1). The rays T_1 and T_2 are parallel and interfere in the telescope. Hence the path difference introduced by rotation is (n being the order of interference)

$$n\lambda = b + R \tan \alpha + (2 R/\cos \alpha) \cos \alpha - (2 R + b - R \tan \alpha) = 2 R \tan \alpha,$$

for the triangle (a,7,2′) is isosceles and its acute angles each α.

The rays T_1 and T_2 have now separated and the amount (4,6) is also $2 R \tan \alpha$. When this exceeds a few millimeters the interferences vanish.

A correction must however be applied, since in the practical apparatus the mirrors rotate at a fixed distance apart. Hence the mirror N_1 must be displaced toward the right (shortening the path) by the normal distance

$$e (R/\cos \alpha - R) \cos 45°$$

and the mirror N_1' toward the left by the same amount. The path difference

introduced is thus a decrement and is twice the $2\,e\cos(45° \mp \alpha)$ of each mirror. Thus the total correction to be subtracted from the equation is after reduction $2R(1-\cos\alpha)(1 \pm \tan\alpha)$. Hence the equation becomes ultimately

$$n\lambda = 2R(\sin\alpha \pm \cos\alpha - 1).$$

To the second order of small quantities, if $i = 45°$ is the angle of incidence, and Δn the normal displacement of M',

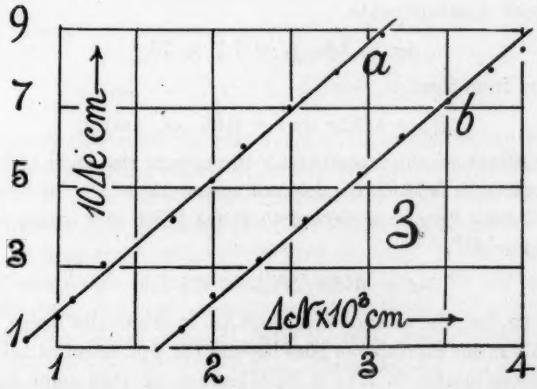
$$\Delta N/\Delta\alpha = R(1 + \Delta\alpha/2)/\cos i.$$

As all the mirrors receive the light on their silvered sides, M originally compensates N if the mirrors are identical in thickness and glass. But the transmission at 3 varies as the angle of incidence changes from $i = 45°$ to $45° - \alpha$. The glass path here decreases by $e(\sin i - \cos i \tan r)\,\Delta\alpha$, where e is the plate thickness, r the angle of refraction. The path difference as above reckoned has thus been increased by this amount and this quantity is to be added to the right hand member. The effect will not usually exceed a few per cent of the air path difference, and the ratio is the same as above.

3. Ocular Micrometer.—It has been stated that the motion of the fringes across the field of the telescope, T, is astonishly swift. Hence it is often desirable to insert a micrometer here, as the displacement of fringes can thus be much more accurately and easily measured than at the micrometer along the normal of the opaque mirror, M', of the interferometer. If the latter is of the type using an auxiliary mirror, *mm*, figure 1, the fringes may even be established of a size to correspond with the ocular micrometer, by rotating the auxiliary mirror; but this is not usually necessary. A good ocular plate micrometer was at hand dividing the width of field (about 1 cm.) into 100 parts, the divisions being 0.1 mm. One-tenth of this is easily estimated by the eye in view of the strong eye lens. The light from the collimator at L should completely fill the field, a condition which may be fulfilled by suitably placing the former, modifying its objective. After completing such preliminary adjustments with the fringes, made very sharp and the ocular scale equally so, this is to be placed at right angles to the fringes. Let Δe denote their displacement measured in centimeters on the ocular scale and ΔN (cm.) the displacement of the opaque mirror M' of the interferometer. The question is whether Δe and ΔN are nearly enough proportional quantities for practical purposes. A number of such standardizations were carried out throughout 1 cm. of Δe, two of which are shown in detail in figure 3. The fluctuation of data is due to air currents across the interferometer. It was not easy to obviate these, and it was not thought necessary for the present purposes. Otherwise the data would have been smooth. There is no doubt that a linear relation may be assumed. In curve *a* the readings of the interferometer micrometer increase, in curve *b* they decrease. If the means be taken from doublets far apart the ratios are

$$(a)\ \Delta N/\Delta e = 0.00310;\ (b)\ \Delta N/\Delta e = 0.00310,$$

and they happen to coincide. Thus Δe is 323 times as large as ΔN and correspondingly easy to measure. The impossibility of setting the micrometer for ΔN accurately enough, since it is graduated to only 5×10^{-5} cm. is completely obviated in Δe. Moreover, as $2\,\Delta N \cos i = \lambda$ (i being the angle of incidence 45°, and λ the mean wave length), we now have $0.0061/\Delta e \cos i = \lambda$; so that the fringe displacement $\Delta e = 0.014$ cm. measured on the ocular micrometer corresponds to the wave length of light in the interferometer measurements. This is more than one scale part. There is however no difficulty in making the fringes larger and obtaining a much more sensitive apparatus in proportion. The achromatic fringes, moreover, when properly produced, contain a distinctive central black line, compatible with the measurement of 0.1 scale part, as here given; i.e. measurement to a few millionths of a centimeter are thus easily feasible under proper surroundings. The apparatus will be used elsewhere.



If $\Delta\varphi$, the angular fringe breadth, is given, $\Delta e/\Delta N$ may be computed from the equation in the earlier paper, to be

$$\Delta e/\Delta N = 2\,L\Delta\varphi \cos i/\lambda$$

or

$$L\Delta\varphi = \lambda/(2 \cos i.\Delta N/\Delta e)$$

as the radius is the length $L = 19.5$ cm. of the telescope. Hence the fringe breadth in centimeters is, if

$$\lambda = 6 \times 10^{-5}\text{cm.}, i = 45°, \text{and } 10^3\Delta N/\Delta e = 3.1,$$
$$L\Delta\varphi = 0.014 \text{ cm.},$$

the value actually observed. Thus if $\Delta\varphi$ is given or measured, $\Delta e/\Delta N$ may be deduced.

The question finally to be determined is thus the value and the meaning of the fringe breadth $\Delta\varphi$. Since

$$2\,\Delta N \cos i = n\lambda$$

if $\Delta N = \Delta N_0$ is constant and also λ, we may then write

$$\Delta\varphi = di/dn = -\lambda/2\,\Delta N_0 \sin i.$$

Furthermore if $\Delta\theta$ is the angular displacement of fringes

$$\Delta\theta = n\Delta\varphi = -(\Delta N/\Delta N_0)\cot i,$$

if $n\lambda$ is replaced by its value and ΔN is small compared with ΔN_0. If it is not, since ΔN_0 involves ΔN, we must state the case thus:

$$-\Delta\theta = \cot i \int_{\Delta N_0}^{\Delta N_0 + \Delta N} dN/dN_0,$$

or on integrating and expanding the natural logarithm

$$-\Delta\theta = \cot i\,[\Delta N/\Delta N_0 - (\Delta N/\Delta N_0)^2/2 + \ldots]$$

and $\Delta e = L\Delta\theta$.

In the above measurements

$$\Delta\varphi = L\Delta\varphi/L = 7.2 \times 10^{-4}$$

whence apart from signs

$$\Delta N_0 = \lambda/2\Delta\varphi \sin i = 0.06 \text{ cm., nearly,}$$

whereas the maximum displacement ΔN throughout the whole series (equivalent to the telescopic field width) does not exceed $\Delta N = 5 \times 10^{-3}$ cm. Hence $(\Delta N/\Delta N_0)^2/2$ may here be neglected to about 1/300 and (again apart from signs) since $i = 45°$,

$$\Delta e = L(\Delta N/\Delta N_0) = 325\,\Delta N,$$

as it should be; i.e., the relation of Δe and ΔN is practically linear, if the displacement ΔN is not excessive or goes beyond the equivalent of field width.

As the determination of ΔN_0 is inconvenient we thus come back to the practical equation already used, or

$$\Delta N/\Delta\theta = \lambda/2\Delta\varphi \cos i,$$

or if $Ld\varphi = \delta e$ and Δe and δe are the fringe displacement and the fringe breadth measured on the same ocular micrometer,

$$\Delta N/\Delta e = \lambda/2\,\delta e \cos i,$$

With this deduction the equations of long distance interferometry, etc., form in terms of δe the fringe breadth and the fringe displacement Δe which may be recorded here, d being the distance,

$$\Delta\alpha = (\lambda/2R\delta e)\Delta e;\; d = (bR\delta e/\lambda)/\Delta e.$$

4. Collimator Micrometer.—For many purposes even better conditions are obtainable by replacing the slit of the collimator by a plate glass micrometer. The magnification in such a case is usually greater and since the telescope now contains no fiducial lines, it need not be fixed, but may be shifted

at pleasure so long as the collimator is fixed. This is often a great convenience in working with the achromatic fringes; but I will pass these details over here.[2]

[1] These PROCEEDINGS, 3, 1917, (563). In this note the glass paths, which are of less importance than the air paths, are too much accentuated.

[2] Abridged from a Report to the Carnegie Institution of Washington, D. C.

THE BRAIN WEIGHT IN RELATION TO THE BODY LENGTH AND ALSO THE PARTITION OF NON-PROTEIN NITROGEN, IN THE BRAIN OF THE GRAY SNAPPER (NEOMAENIS GRISEUS)

BY SHINKISHI HATAI

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Communicated by A. G. Mayer, December 24, 1917

The predatory fish called the gray snapper, *N. griseus* was mainly used for the present investigation, which was conducted at Tortugas, Florida in the summer of 1917. The following are the more important facts brought out.

1. The Brain Weight in Relation to Body Length.—Altogether observations have been made upon 74 brains of the gray snapper. It was found that the relation of brain weight to the increasing body length, from 150 mm. upward, is practically linear and may be satisfactorily expressed as $y = a + bx$, where y represents brain weight in grams and x the body length in millimeters, and a and b are the constants with the values, in this instance, -0.333 and 0.00433 respectively. (Body length is measured from the tip of the snout to the crotch of the tail.)

It is well known that in the adult stage the relation between brain weight and body length or body weight is practically linear, even in the case of some mammals[1] but it is remarkable to find the linear relation in fish, when they are so small. This linear relation during the period of early growth probably means that in the snapper the brain reaches its structural maturity early, and that the subsequent increase in weight indicates merely a uniform swelling of the nervous tissue as a whole.

On account of scantiness of the data for specimens less than 200 mm. in body length, I am unable to present a complete record of the growth of the brain. However, it appears from the general trend of the growth-curve that with the possible exception of the very early period, the relation between the brain weight and body length is linear.

Kellicott[2] studied the growth of the brain in the smooth dogfish in respect to the body weight, and found the graph to resemble that of the mammalian brain (logarithmic curve) and thus to differ strikingly from that for the gray

snapper. This difference may be due to the fact that in the dogfish the brain possesses a voluminous cerebellum, as well as olfactory bulbs, and the combined weights of these two structures being greater than that of the rest of the brain, while these two structures in the gray snapper are very small. It appears that these two parts, olfactory bulbs and cerebellum of the dogfish brain, grow very rapidly during the earlier period, thus giving a form of the graph similar to that of the mammal.

2. Percentage of Water in the Brain.—Altogether 64 snappers were examined. It was found that so far as the present data are concerned, the percentage of water in the small and large fish is nearly identical within a wide range of body length, and therefore the percentage of water does not vary regularly with the length or size of the fish. Similar relations were observed by Donaldson[3] in the brain of the summer flounder and by Scott[4] in the brain of the smooth dogfish.

It is also interesting to note that the average percentages of water obtained by Donaldson, Scott and myself are nearly the same, being 78.4% (flounder), 78.5% (dogfish) and 78.61% (gray snapper) respectively. It is worthy of note that the values given by these fish are not much different from (slightly above) the percentage of water in the adult mammalian brain.

Since the reduction of the water in the brain is induced by the deposition of the 'myelin substance'[5] we may infer that the process of myelination in the fish brain occurs at a very early period, thus producing slight variation from small to large individuals, as here recorded. I have not however been able to determine the period of rapid reduction of the water content in the brain which must take place in this fish in consequence of the appearance of myelin, because of lack of very young material.

3. Chemical Analysis of the Brain.—Altogether 51 dried brains as well as 44 fresh brains were used for the purpose of the chemical analysis. With respect to the nitrogen in total solids, nitrogen in ether-alcohol extract, and the lipoid content, the fish brain closely resembles the stem of the rat brain, but significantly differs from the entire rat brain. This is explained by the fact that the fish brain corresponds essentially to the stem of the mammalian brain, owing to the small growth of the cerebrum and cerebellum. The content of non-protein nitrogen is considerably greater in the fish than in the rat brain. This phenomenon is interesting and I wish to call attention to two factors which may have some bearing on it.

It seems probable that on account of the low grade of organization of the fish brain, the physical consistence of the nervous system may not be as stable as that of more highly organized mammalian nervous systems, and thus the wear and tear process may be much greater, producing a correspondingly greater amount of waste products in the fish brain.

Again it has been found by several investigators that fish blood contains a greater abundance of non-protein nitrogen compared with human blood. The content of non-protein nitrogen in the rat blood is nearly identical with that

in human blood.[6] It seems therefore conceivable that the high content of the non-protein nitrogen in the fish blood, accompanied by a slow circulation, might favor the deposition of waste products, and at the same time their slow removal would further tend to increase the accumulation.

[1] Donaldson, H. H., *J. Comp. Neur., Wistar Inst., Philadelphia*, **19**, 1909, (155–167).
[2] Kellicott, W. E., *Amer. J. Anat., Wistar Inst., Philadelphia*, **8**, 1908, (319–353).
[3] Donaldson, H. H., On the percentage of water in the brain of the summer flounder (*Paralichthys dentatus*) according to the body weight. MS. 1905.
[4] Scott, G. G., *Proc. Soc. Exp. Biol. Med., New York*, **9**, 1912.
[5] Donaldson, H. H., *J. Comp. Neur., Wistar Inst., Philadelphia*, **26**, 1916, (443–451).
[6] Hatai, S., *Ibid.*, **28**, 1917, (361–378).

THE ROTATION AND RADIAL VELOCITY OF THE CENTRAL PART OF THE ANDROMEDA NEBULA

By F. G. Pease

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Communicated by G. E. Hale, December 27, 1917

Indications of rotation in spiral nebulae are quickly obtained by means of spectrographs of very short focus and small dispersion, especially when a wide slit and devices for narrowing the spectrum are used. For definitive values of the rotation, however, it is necessary that the scale be relatively large, the slit narrow, and the exposures very long.

To investigate the rotation of the great nebula in Andromeda, which had been reported by V. M. Slipher at the nineteenth meeting of the American Astronomical Society, an exposure of seventy-nine hours was made during August, September, and October, 1917, with the focal-plane spectrograph of the 60-inch reflector. This instrument which is used at the primary focus of the large mirror has a collimator of 135 mm. focus, a 60° prism of ultra-violet glass, and for the camera a 154 mm. Cooke lens. The dispersion is such that 5.3 mm. corresponds to the interval from λ 4930 to λ 4950. The ratio of focal lengths of camera and collimator is 1.14, and the spectrograph therefore magnifies the image on the slit by this amount.

TABLE 1

DISTANCE FROM NUCLEUS		NUMBER OF SECTIONS	ADAMS	MISS BURWELL	MEAN
seconds			*km.*	*km.*	*km.*
South Prec.	162	1	(−355)	(−469)	(−412)
	121	2	369	400	385
	66	2	342	342	342
	25	1	310	312	311
	0	1	307	317	312
North Foll.	41	2	288	308	298
	96	2	278	288	283
	137	1	(292)	(257)	(274)

During the exposure the slit crossed the nucleus and coincided with the major axis which lies in position angle 40°. The slit-width was 0.025 mm., and the spectrogram was traversed longitudinally by comparison spectra at intervals of 1 mm. (see fig. 1).

The results of measures by Mr. Adams and Miss Burwell are given in table 1. The correction for the earth's motion is slight and has been neglected. To reduce the accidental errors and equalize the weights somewhat, results for adjacent sections of spectra in the outer regions have been combined. The values for extreme sections, which are very uncertain, are given separately and appear in parentheses in the table. The values of the distance from the nucleus have been corrected for magnification by the spectrograph. The tabulated radial velocities must be corrected for the inclination of the nebula to the line of sight. Assuming the nebula to be circular, the ratio of the major and minor axes of its apparent elliptical outline, which is 4 to 1, is a measure of the inclination. We thus find approximately 14° as the inclination, and the velocities must accordingly be multiplied by 1.03 to reduce them to the plane of the nebula.

The mean velocities of table 1 are shown in figure 2, the ordinates being kilometers per second and the abscissae seconds of arc. The velocity curve

$$y = -0.48\,x - 316$$

was determined graphically. From this equation we find:

1. The radial velocity of the nebula is −316 km., a value in good agreement with the earlier measures:

	km.
Pease and Adams[1]	−329
Pease and Adams[2]	−297
Slipher[3]	−300
Wolf[3]	−350
Wright[3]	−304

2. The linear velocity of rotation at a point 2 minutes of arc from the nucleus is 58 per second.

3. Within the distance measured and the limits of accuracy of the measures the change of rotational velocity with distance seems to be linear, although there may be variations at individual points in the nebula.

4. Whether the motion of the nebula is inward or outward along the arms of the spiral depends upon the inclination of the nebula.

From a spectrogram of eighty-four hours exposure made at intervals during August, September, and October, 1916, with the slit placed on the minor axis of the nebula, Mr. Adams obtains the results shown in table 2.

These velocities, which are nearly constant, are also shown in figure 2 the velocity curve obtained for points along the major axis, and greatly with

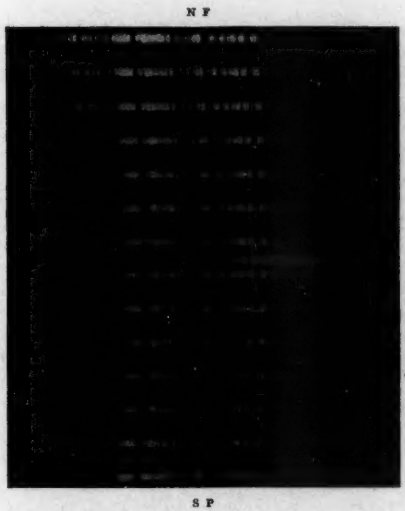

FIG. 1. SPECTRUM OF THE CENTRAL PARTS OF THE ANDROMEDA NEBULA. EXPOSURE 79 HOURS

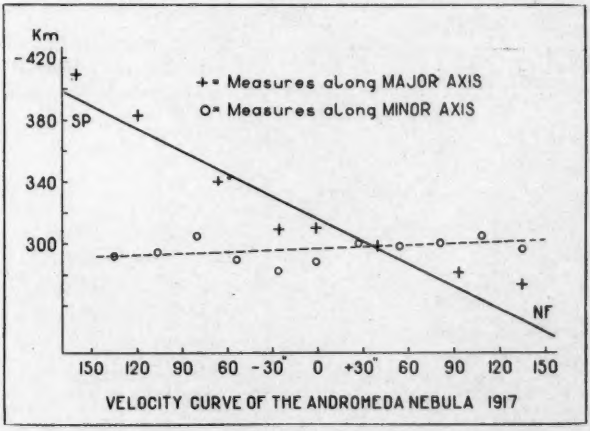

FIG 2

strengthen the conclusions derived from the latter by showing that the variable velocity given by the spectrogram first described is without doubt to be attributed to the rotation of the nebula.

The arms of the cross in figure 3 indicate the length and position of the slit on the nebula.

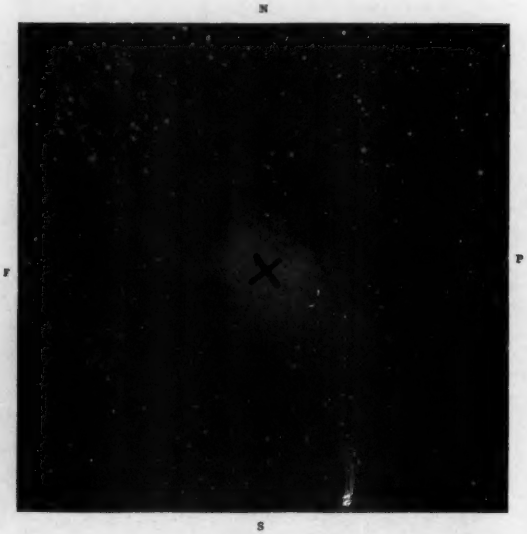

FIG. 3. DIAGRAM SHOWING THE LENGTH AND POSITIONS OF SLIT ON THE NEBULA

TABLE 2

SECTION OF SPECTRUM	RADIAL VELOCITY	NUMBER OF LINES
	km.	
5th, below	−293	3
4th	297	4
3rd	306	7
2nd	291	5
1st	284	6
Nucleus	291	9
1st, above	300	8
2nd	299	6
3rd	302	6
4th	306	4
5th	−297	5
Mean	−297	

Both spectrograms show a solar-type spectrum entirely free from bright lines. Mr. W. P. Hoge assisted in the guiding during the long exposures.

[1] Pease, F. G., *Pub. Astr. Soc. Pac., San Francisco, Cal.*, **27**, 1915, (134).
[2] This paper following.
[3] Slipher, V. M., *Pop. Astr., Northfield, Minn.*, **23**, 1915, (21–24), page 23.